**Research article**

# MATHEMATICAL MODEL TO MONITOR THE RATE OF DISSOLVED URANIUM INFLUENCED BY VOID RATIO AND LINEAR VELOCITY IN HOMOGENEOUS COARSE SAND IN ELELE, RIVERS STATE OF NIGERIA

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## Abstract

Mathematical model to monitor the rate of dissolved uranium influenced by void ratio and linear velocity in homogeneous coarse sand has been thoroughly expressed. An application of mathematical tool was used to express the migration and deposition of uranium in soil and water environment. The study area is an industrialized location whereby the activity of man generates lots of activities in the study area. Formation characteristics were found to influence the deposition and migration of uranium, base on theses factors, mathematical expression were imperative to develop a mathematical model that will monitor the deposition and migration from this dimension. The study considered the deposition and action on uranium in various dimension. Uranium is a naturally occurring radioactive element that is present in nearly all rocks and soils; it has an average concentration. Uranium can undergo oxidation-reduction reactions in the environment or microbial reactions to form complexes with organic matter. Uranium may be redistributed in the environment by both anthropogenic and natural processes. The three primary industrial processes that cause this redistribution are operations associated with the nuclear fuel cycle that include the mining, milling, and processing of uranium ores or uranium end products; the production of phosphate fertilizers for which the phosphorus is extracted from phosphate rocks containing uranium; and the improper disposal of uranium mine tailings. Uranium deposited by wet or dry precipitation will be deposited on land or in surface waters. If land deposition occurs, the uranium can be reincorporated into soil. **Copyright © IJWMT, all rights reserved.**

**Keywords:** mathematical model, dissolved uranium, void ratio and linear velocity

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## 1. Introduction

Metals contamination is a persistent problem at many contaminated sites. In the U.S., the most commonly occurring metals at Superfund sites are lead, chromium, arsenic, zinc cadmium, copper, and mercury. The presence of metals in groundwater and soils can pose a significant threat to human health and ecological systems. The chemical form of the metal contaminant influences its solubility, mobility, and toxicity in ground-water systems. The chemical form of metals depends on the source of the metal waste and the soil and ground-water chemistry at the site. A detailed site characterization must be performed to assess the type and level of metals present and allow evaluation of remedial alternatives. A number of the available technologies have been demonstrated in full-scale applications and are presently commercially available. A comprehensive list of these technologies is available (U.S. EPA, 1996a). Several other technologies are being tested for application to metals-contaminated sites Treatment of metals-contaminated groundwater has typically involved flushing and aboveground treatment, while treatment of contaminated solids most often has been performed by excavation followed by ex situ treatment or disposal. The most common ex situ treatment for excavated soils is solidification/stabilization. Soil consists of a mixture of weathered minerals and varying amounts of organic matter. Soils can be contaminated as a result of spills or direct contact with contaminated waste streams such as airborne emissions, process solid wastes, sludges, or leachate from waste materials. The solubility of metals in soil is influenced by the chemistry of the soil and groundwater (Sposito, 1989; Evans, 1989). Factors such as pH, Eh, ion exchange capacity, and complexation/chelation with organic matter directly affect metal solubility.

Surface water and groundwater may be contaminated with metals from wastewater discharges or by direct contact with metals-contaminated soils, sludges, mining wastes, and debris. Metal-bearing solids at contaminated sites can originate from a wide variety of sources in the form of airborne emissions, process solid wastes, sludges or spills. The contaminant sources influence the heterogeneity of contaminated sites on a macroscopic and microscopic scale. Variations in contaminant concentration and matrix influence the risks associated with metal contamination and treatment options.

Most published research reports have been focused on bioreduction of U(VI) by various microbial cultures at laboratory scale (e.g., Lovley et al., 1991; Lovley and Phillips, 1992a,b; Gorby and Lovley, 1992; Ganesh et al., 1997; Truex et al., 1997; Abdelouas et al., 1998; Fredrickson et al., 2000; Fredrickson et al., 2002; Holmes et al., 2002). Kinetics have been analyzed for defined or mixed cultures in laboratory (e.g., Liger et al., 1999; Spear et al., 1999, 2000). Under field conditions, U(VI) undergoes hydrological, geochemical, and biological processes in complex interaction, such as sorption/desorption, advective-dispersive transport, and

microbial transformations. Uranium sorption/desorption is significantly influenced by bicarbonate concentrations and pH (Waite et al., 1994; Wazne et al., 2003). At the sorption sites, uranium competes with other ions. Since the geochemical environment may vary over the course of the experiment, simplified approaches to model U(VI) sorption, such as the assumption of a linear retardation factor, appear insufficient (Bain et al., 2001). For bioreduction of U(VI), nitrate, Fe (III) and sulfate serve as competing electron acceptors which should be considered in the simulations (e.g., Wielinga et al., 2000; North et al., 2004; Wu et al., 2005). In the presence of significant

calcium concentrations, the highly stable but poorly biodegradable calcium–uranyl– carbonate complexes should also be included in the simulation (Bernhard et al., 1996; Kalmykov and Choppin, 2000; Bernhard et al., 2001; Brooks et al., 2003).

## 3. Governing Equation

$$V \frac{i\partial^2 C}{\partial t^2} = - \left[ \frac{Kj\varepsilon p}{\mu} \right] \left[ V \frac{\partial C}{\partial x} + Pg \frac{\partial C}{\partial xi} \right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (1)$$

The expressed governing equations are modified as a system that expresses the deposition of uranium in soil and water environment. This system was modified through various parameters that are influential to the transport of uranium in soil structural deposition in the study area. In line with the structural composition of the formation, it became necessary that parameters that expressed fast migration of these contaminants under the influence of formation characteristics should be considered. This modified the expressed governing equation as stated above.

Taking the Laplace transformation of (1)

$$\frac{\partial^2 C}{\partial t^2} = SC_{(0)} - SC_{(x)} - C_{(0)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (2)$$

$$\frac{\partial C}{\partial x} = S^1 C_{(x)} - SC_{(x)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

$$C_{(x)} = C_{(0)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (4)$$

Substituting equation (2), (3) and (4) into equation (1) yields

$$Vi \left[ S^2 C_{(x)} - SC_{(x)} + C_{(0)} \right] + \frac{Kj\varepsilon p}{\mu} + \left[ VSC_{(x)} - Pg\,SC_{(x)} \right] C_{(0)} \qquad \ldots\ldots\ldots\ldots \qquad (5)$$

$$Vi\,S^2 C_{(t)} - ViS^1 C^1{}_{(t)} + C_{(0)} + \frac{Kj\varepsilon p}{\mu} C_{(0)} + C_{(0)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (6)$$

The derived solutions were under continuous expression in line with the influential variables that structure the system, this is to express the migration process of deposited uranium in soil and water environment. Based on these variables, it is imperative to determine their boundary values that express their limit of migration under the structured stratification of the strata as influenced by the geologic history of the formation. These conditions were considered in developing the boundary values that soothe the structured stratification under the influence of formation characteristics.

Considering the following boundary condition at $t = 0$, $C^1{}_{(0)} = C_0 = 0$ …….. $\qquad$ (7)

We have

$$C_{(t)} \left( Vi\ S^2 - Vs + \frac{Kj\varepsilon p}{\mu} + V + Pg \right) = 0 \qquad \text{................................} \qquad (8)$$

$$C_{(t)} \neq 0 \qquad \text{..............................} \qquad (9)$$

Following the expressed solution by considering further boundary values where $t > 0$ influencing the concentration, these were possible based on the variation of the structural deposition that influence uranium concentration in a particular region on transport process. The expressed boundary values are developed to monitor the time of migration under the change of concentration with respect to time. Subject to this relation, the migration process of uranium change in concentration are deposited depending on the strata micropores, this deposition of uranium detailed the intercalation of the minerals in the particle grain size structured under the influence of disintegration of the sedimentary deposit in the formation. These conditions are noted in considering boundary values that express the time of transport influencing the concentration of uranium in soil and water environment.

But considering the boundary condition

At $t > 0,\ C^1_{(0)} = C_{(0)} = C_o \qquad \text{...............................} \qquad (10)$

$$S^2 C_{(t)} - \frac{Kj\varepsilon p}{\mu} SC_{(x)} + Vs\ S\ C_{(x)} = Vi\ Sc_o + Vi\ C_o + \frac{Kj\varepsilon p}{\mu} C_o + Pg\ C_o \qquad \text{......} \qquad (11)$$

$$\left[ Vi\ S^2 - \frac{Kj\varepsilon p}{\mu} + P + Pg \right] C_{(t)} = \left[ Vi\ S + Vi + \frac{Kj\varepsilon p}{\mu} + V + Pg \right] C_o \qquad \text{............} \qquad (12)$$

$$C_{(x)} = \frac{Vi\ S - ViS + \dfrac{Kj\varepsilon p}{\mu} + V + Pg}{Vi\ S^2 - \dfrac{Kj\varepsilon p}{\mu} + V + Pg}\ C_o \qquad \text{.....................} \qquad (13)$$

Further expressions on the derived solution, application of quadratic function were found suitable to develop or establish a relationship; this is to express the functions between both variables as it is modified in the system. This condition implies that quadratic functions will definitely express their relations as they are influential in the system to develop a conceptual structure that influence the influential variables in the system, it were structured in the formation of the developed governing equation. This implies that the system will definitely showcase the expressed functions to detail the structure of uranium migration at different formations under the influence of heterogeneous and homogeneous structural stratification of the formation in soil and water environment.

Applying quadratic expression, we have

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$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{(14)}$$

Where $a = Vi$, $b = \dfrac{Kj\varepsilon p}{\mu}$, $c\ VPg$

$$-\frac{\dfrac{Kj\varepsilon p}{\mu} \pm \sqrt{\dfrac{Kj\varepsilon p}{\mu}^2 + 4ViVPg}}{2Vi} \qquad \text{(15)}$$

$$C_{(t)} = A\exp\left[\frac{-\dfrac{Kj\varepsilon p}{\mu} + \sqrt{\dfrac{Kj\varepsilon p}{\mu}^2 + 4ViVPg}}{2Vi}\right]^t - \exp\left[\frac{-\dfrac{Kj\varepsilon p}{\mu} + \sqrt{\dfrac{Kj\varepsilon p}{\mu}^2 + 4ViVPg}}{2Vi}\right]^t \quad \text{(16)}$$

Subjecting equation (16) to the following boundary condition and initial values condition

The rate of uranium deposition in soil are express in this mathematical condition that are applied as state in the derived solution, the rate of accumulation under the influences of regeneration of the solute in the study location, the stated condition implies that the system are considered in different dimension base on the variation in structural deposition of the formation, base on this circumstances the stated boundary values are considered in other determined the rate of concentrations in this dimension, thus expressed in the considered boundary values in equation [7].

$$x = 0,\ t > 0,\ C_0 = 0 \qquad \text{(17)}$$

We have $B = -1$ and $A = 1$ ............................... (18)

So that our particular solution we have it this form

$$C_{(t)} = \exp\left[-\frac{Kj\varepsilon p}{\mu} + \left(\frac{Kj\varepsilon p}{\mu}^2 - 4ViVPg\right)^{\frac{1}{2}}\right]^t - \exp\left[-\frac{Kj\varepsilon p}{\mu} + \left(\frac{Kj\varepsilon p}{\mu}^2 + 4ViVPg\right)^{\frac{1}{2}}\right] \quad \text{(19)}$$

The expressed condition in equation [19] show the exponential phase of the system under the influences of accumulation of the contaminant in the transport process, such condition implies that the rate of accumulation are develop through the rate of regeneration of industrial waste in the study location, such expression implies that the formation characteristics express the rate of migration from one formation to the other under the influences of plug flow, therefore the exponential rate of the system are determined by these applied condition .

But $e^x + e^{-x} = 2Sin\ x$

These is the tendency of variation in deposition of uranium in the formation , therefore variation of the stratification implies that the migration or deposition will not be homogeneous in the system, therefore the condition of uranium deposition in exponential phase to the level of fluctuation will be possible in soil and water environment, such expression implies that the stratification will reflect the deposition of uranium, it means that the concentration will

not be homogeneous, base on this condition application of the function above become suitable in deposition and migration of dissolved uranium.

Therefore, the model can be expressed as;

$$C_{(t)} = 2Sin\left[\frac{Kj\varepsilon p}{\mu} + \left(\frac{Kj\varepsilon p}{\mu}^2 + 4ViVPg\right)^{\frac{1}{2}}\right]^{\frac{d}{v}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (20)$$

But if $t = \dfrac{d}{v}$

Therefore, the model can be expressed as:

$$C_{(x)} = 2Sin\left[\frac{Kj\varepsilon p}{\mu} + \left(\frac{Kj\varepsilon p}{\mu}^2 + 4ViVPg\right)^{\frac{1}{2}}\right]^{\frac{d}{v}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (21)$$

Again, if $\dfrac{v}{t} = x,$ we have

$$C_{(t)} = 2Sin\left[\frac{Kj\varepsilon p}{\mu}\left(\frac{Kj\varepsilon p}{\mu}^2 + 4ViVPg\right)^{\frac{1}{2}}\right]^{t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (21)$$

Considering (21) and (22) yield

$$C_{(x,t)} = 2Sin\left[\frac{Kj\varepsilon p}{\mu} + \left(\frac{Kj\varepsilon p}{\mu}^2 + 4ViVPg\right)^{\frac{1}{2}}\right]^{t} + 2Sin\left[\frac{Kj\varepsilon p}{\mu} + \left(\frac{Kj\varepsilon p}{\mu}^2 + 4ViVPg\right)^{\frac{1}{2}}\right]^{x} \qquad (22)$$

The expression in [22] is the developed model that will monitor the deposition and migration of dissolved uranium in soil and water environment. The development of this model was through the expressed parameters in the system that are found to influence the migration and deposition of uranium in the study location, subject to this relation, the rate of formation variation as an influential parameters was base on the deposition and transport system of dissolved uranium such expression implies that the stratification history are structured from the geologic history which is considered on the development of the system that generate the governing equation.

## 4. Conclusion

Uranium been identified in at least 67 of the 1,699 hazardous waste sites have been proposed for inclusion on the EPA National Priorities. However, the number of sites evaluated for uranium is not known. Therefore, the activity

of uranium remains essentially unchanged over periods of thousands of years. Uranium may be redistributed in the environment by both anthropogenic and natural processes. Uranium can undergo oxidation-reduction reactions in the environment or microbial reactions to form complexes with organic matter

Uranium deposited by wet or dry precipitation will be deposited on land or in surface waters. If land deposition occurs, the uranium can be reincorporated into soil, resuspended in the atmosphere (typically factors are around $10^{-6}$), washed from the land into surface water, incorporated into groundwater, or deposited on or adsorbed onto plant roots (little or none enters the plant through leaves or roots). Conditions that increase the rate of formation of soluble complexes and decrease the rate of sorption of labile uranium in soil and sediment enhance the mobility of uranium. Base these conditions mathematical model was find imperative to express the behaviour of it deposition and migration in soil and water environment. The study was to showcase the behaviuor of the contaminant in terms of migration.

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